Work, Energy and Power

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Note : The notes given in this file is no substitute to the much detailed discussion held in the online/contact classes with active participation of students. It , at best, serves the purpose of ready reference for important concepts/derivations covered in the classes.

Kinetic energy

It is the energy possessed by a body by the virtue of its motion.

$$\boxed{KE = \frac{1}{2}mv^2}$$

$$\boxed{KE = \frac{p^2}{2m}}$$

- Kinetic energy is a scalar quantity
- SI unit of kinetic energy is joule (J)
- Kinetic energy can be either zero (for an object at rest) or positive (for an object in motion). *KE* can never be negative.

Potential energy

It is the energy possessed by a system by the virtue of its position or state or configuration.

- Potential energy is a scalar quantity
- SI unit of potential energy is joule (J)
- Potential energy may be positive, negative or zero
- Various expressions for potential energy arise due to various types of forces associated with a phenomenon
- Potential energy is associated with only that kind of force for which the work done by the force is stored in the system in some form and again manifests itself as kinetic energy once the constraints are removed

Conservative forces

- Work done by the force in a closed path is zero
- Work done by the force is independent of the path followed and depends only on initial and final positions
- Force is derivable as a negative gradient of scalar potential energy
- Examples : gravitational force, spring force, electrostatic force

Non-conservative forces

- Work done by the force in a closed path is not zero
- Work done by the force is depends on the path followed by the body
- Force is not derivable as a negative gradient of scalar potential energy
- Examples : frictional force, viscous force

Potential energy (examples)

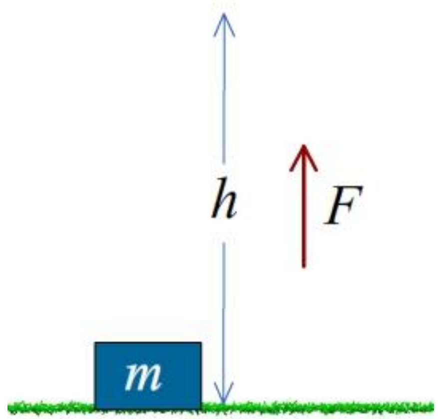
Potential energy of a body raised from ground

Consider a body of mass m on the ground. Let the body be lifted, very slowly, to a height h from the ground. Force required to lift the body (i.e. to overcome the gravitational force on the body) is mg (upwards) therefore work done by the applied force in lifting the body up is

$$W = mg \times h \times \cos(0)$$

$$W = mgh \times 1$$

$$W = mgh$$



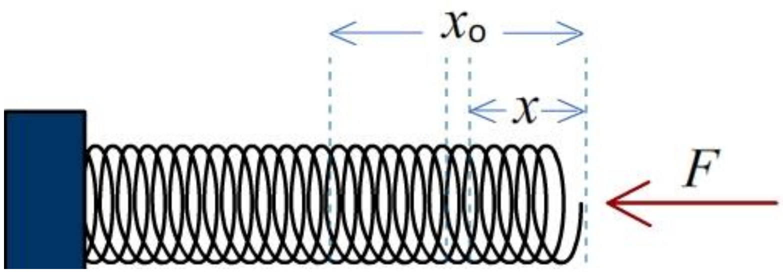
Work done by the external agent on the body is stored in the form of potential energy in it.

$$\boxed{PE_G = mgh}$$

The fact that work done is stored as energy can be verified by leaving the body from that height. The body gains kinetic energy as it reaches the ground !

Potential energy (examples)

Potential energy stored in a spring



Consider a spring of spring constant k. From the initial relaxed state the spring is compressed through a distance x_o. Work in causing a small compression (dx) is

$$\mathrm{d}W = F \times \mathrm{d}x \times \cos(0)$$

$$\mathrm{d}W = F\ \mathrm{d}x$$

Force applied externally (to overcome the restoring force by the spring) is kx.

$$\mathrm{d}W = kx\ \mathrm{d}x$$

Total work done by external force in causing a compression of x_o is

$$\int \mathrm{d}W = \int_0^{x_o} kx\ \mathrm{d}x$$

$$W = k\int_0^{x_o} x^1\ \mathrm{d}x$$

$$W = k\left[\frac{x^2}{2}\right]_0^{x_o}$$

$$W = k\left[\frac{x_o^{\ 2}}{2} - \frac{0}{2}\right]$$

This work done is stored as *PE* in the spring

$$\boxed{PE = \frac{1}{2}kx_o^{\ 2}}$$

Work energy theorem

Work done by all the forces acting on a body is equal to the change in its kinetic energy.

$$\boxed{W_{\text{all}} = \Delta KE}$$

$$\boxed{W_{\text{ext}} + W_{\text{INC}} + W_{\text{IC}} = \Delta KE}$$

W_{ext} : Work done by external force

W_{INC} : Work done by internal non conservative force

W_{IC} : Work done by internal conservative force

Work done by internal conservative forces is also equal to negative of change in potential energy ($-\Delta PE$). Therefore

$$W_{\text{ext}} + W_{\text{INC}} - \Delta PE = \Delta KE$$

$$\boxed{W_{\text{ext}} + W_{\text{INC}} = \Delta KE + \Delta PE}$$

Work energy theorem (for constant force)

Consider a body of mass m having an initial velocity u. Under the action of a constant force (F) let the body attain a final velocity v as it undergoes a displacement S.

Using the equation of motion for a body under constant acceleration we get

$$v^2 - u^2 = 2aS$$

Multiplying both sides of the above equation with $m/2$ we get

$$\frac{m}{2}v^2 - \frac{m}{2}u^2 = \frac{m}{2} \times 2aS$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = maS$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = FS$$

$\frac{1}{2}mv^2$ is final kinetic energy

$\frac{1}{2}mu^2$ is initial kinetic energy

$F\ S$ is work done by the force

therefore

$$KE_{\text{f}} - KE_{\text{i}} = W$$

$$\boxed{W = \Delta KE}$$

Work energy theorem (for variable force)

Consider a body of mass m having an initial velocity u. Under the action of a force let the body attain a final velocity v.

Consider change in kinetic energy of a body as a function of time

$$\frac{\mathrm{d}KE}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}mv^2\right)$$

Assuming mass to be constant we get

$$\frac{\mathrm{d}KE}{\mathrm{d}t} = \frac{1}{2}m\frac{\mathrm{d}}{\mathrm{d}t}\left(v^2\right)$$

$$\frac{\mathrm{d}KE}{\mathrm{d}t} = \frac{1}{2}m2v\frac{\mathrm{d}v}{\mathrm{d}t}$$

$$\frac{\mathrm{d}KE}{\mathrm{d}t} = mv\frac{\mathrm{d}v}{\mathrm{d}t}$$

$$\frac{\mathrm{d}KE}{\mathrm{d}t} = ma\,v$$

$$\mathrm{d}KE = F\ \mathrm{d}x$$

To obtain total change in kinetic energy we integrate the above expression

$$\int_{\mathrm{i}}^{\mathrm{f}} \mathrm{d}KE = \int F\ \mathrm{d}x$$

$$KE_{\mathrm{f}} - KE_{\mathrm{i}} = W$$

$$\boxed{W = \Delta KE}$$

Law of conservation of energy

If internal forces are conservative in nature and external forces do no work on the system then total energy of the system remains constant.

From the work-energy theorem we get

$$W_{\text{ext}} + W_{\text{INC}} + W_{\text{IC}} = \Delta KE$$

When work done by the external forces is zero we get

$$W_{\text{INC}} + W_{\text{IC}} = \Delta KE$$

When internal forces are conservative then work done by them is given by – ΔPE therefore

$$-\Delta PE = \Delta KE$$

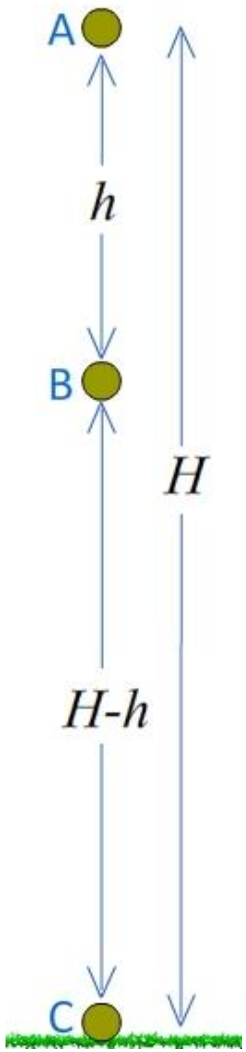
$$\Delta KE + \Delta PE = 0$$

Law of conservation of energy - freely falling body

Consider a body of mass m dropped from a height H above the ground.

As the body reaches the ground its kinetic energy increases and its potential energy decreases.

Consider three points A, B and C along its path of its descent , as shown in the figure.



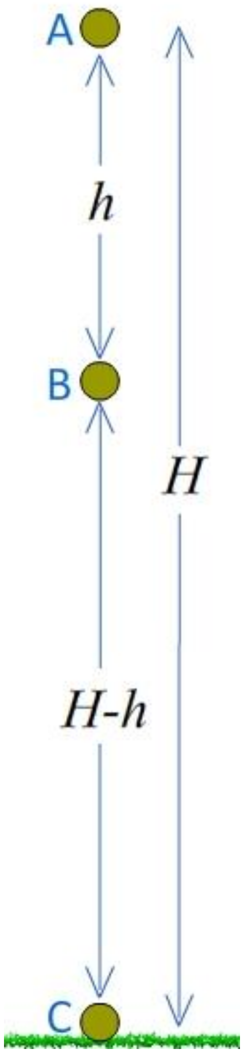
At point A

$$TE_A = KE_A + PE_A$$

$$TE_A = 0 + PE_A$$

$$TE_A = mgH \quad \text{i}$$

Law of conservation of energy - freely falling body



At point B

$$TE_B = KE_B + PE_B$$

$$TE_B = \frac{1}{2} m v_B^2 + mg(H-h) \quad \boxed{\text{ii}}$$

considering motion from A to B

$$v^2 - u^2 = 2aS$$

$$v_B^2 = 2gh \quad \boxed{\text{iii}}$$

Substituting this in eq (ii)

$$TE_B = \frac{1}{2} m 2gh + mg(H-h)$$

$$TE_B = mgH \quad \boxed{\text{iv}}$$

At point C

$$TE_C = KE_C + PE_C$$

$$TE_C = \frac{1}{2} m v_C^2 + 0 \quad \boxed{\text{v}}$$

considering motion from A to C

$$v^2 - u^2 = 2aS$$

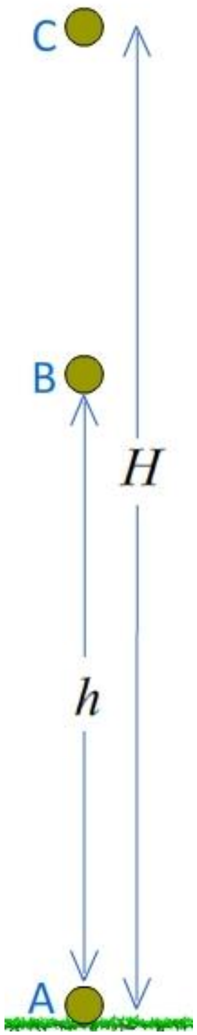
$$v_C^2 = 2gH \quad \boxed{\text{vi}}$$

Substituting this in eq (v)

$$TE_C = mgH \quad \boxed{\text{vii}}$$

From eqs (i), (iv) and (vii) it is verified that total energy of a freely falling body remains constant

Law of conservation of energy – vertically projected body



Consider a body of mass m projected vertically up with an initial velocity u.

As the body ascends its kinetic energy decreases and its potential energy increases.

Consider three points A, B and C along the path of its ascent, as shown in the figure.

At point A

$$TE_A = KE_A + PE_A$$

$$TE_A = \frac{1}{2}mu^2 + 0$$

$$TE_A = \frac{1}{2}mu^2 \qquad \text{i}$$

Law of conservation of energy – vertically projected body

At point B

$$TE_B = KE_B + PE_B$$

$$TE_B = \frac{1}{2} m v_B{}^2 + mgh \quad \boxed{\text{ii}}$$

considering motion from A to B

$$v^2 - u^2 = 2aS$$

$$v_B{}^2 - u^2 = 2(-g)h$$

$$h = \frac{(u^2 - v_B{}^2)}{2g} \quad \boxed{\text{iii}}$$

Substituting this in eq (ii)

$$TE_B = \frac{1}{2} m v_B{}^2 + mg \frac{(u^2 - v_B{}^2)}{2g}$$

$$TE_B = \frac{1}{2} m u^2 \quad \boxed{\text{iv}}$$

At point C

$$TE_C = KE_C + PE_C$$

$$TE_C = 0 + mgH \quad \boxed{\text{v}}$$

considering motion from A to C

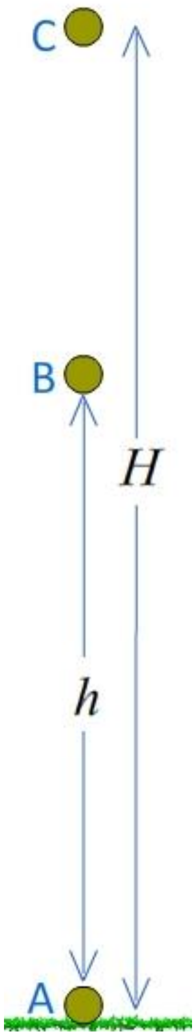
$$v^2 - u^2 = 2aS$$

$$0 - u^2 = 2(-g)H$$

$$H = \frac{u^2}{2g} \quad \boxed{\text{vi}}$$

Substituting this in eq (v)

$$TE_C = \frac{1}{2} m u^2 \quad \boxed{\text{vii}}$$



From eqs (i), (iv) and (vii) it is verified that total energy of a vertically projected body remains constant